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The influence of Anderson localization on the mode decay of excited nonlinear systems

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Abstract. A one-dimensional system of masses with nearest-neighbor interactions and periodic boundary conditions is used to study mode decay and ergodicity in nonlinear, disordered systems. The system is given an initial periodic displacement, and the total system energy within a specific frequency channel is measured as a function of time. Results indicate that the rate of mode decay at early times increases when impurities are added. However, for long times the rate of mode decay decreases with increasing impurity mass and impurity concentration. This behavior at long times can be explained by Anderson localization effects and the nonergodic response of the system.

Keywords: Anderson localization, disordered systems, nonlinear systems
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1 Introduction

The transition from quasi-periodic to ergodic behavior in nonlinear systems has been an active field of research since the seminal work of Fermi, Pasta, and Ulam (FPU) [1]. The behavior of the FPU system has been discussed in great detail [2, 3], and given the degree of nonlinearity and initial energy density, one can estimate the time required before the system becomes ergodic. However, little is known about the effects that impurities have upon the transition to ergodic behavior in nonlinear systems. Since mode coupling in nonlinear systems occurs through interactions, one expects that the presence of impurities will hasten the transition to ergodic behavior. This expectation is, however, in general not correct due to Anderson localization effects [4].

Reported here are the results from a numerical experiment using a one-dimensional system of masses with nonlinear nearest neighbor forces and periodic boundary conditions. The displacement of each mass is sampled over a finite interval of time and the energy within all frequencies is calculated for each mass. The total energy in a single mode is conserved in harmonic systems both with and without impurities. Therefore, for the nonlinear systems, the effects of the impurities upon the mode decay can be compared directly to the systems without impurities.

2 Numerical experiment

The system used in this experiment, which corresponds to the FPU β-model, is composed of N masses with periodic boundary conditions, and with unit equilibrium spacing. The masses undergo a displacement $u(t)$, and the i-th mass m_i interacts through nearest neighbor forces:

$$m_i\ddot{u}_i = -\kappa\left[(u_{i+1} - u_i) - (u_i - u_{i-1})\right] - \beta\left[(u_{i+1} - u_i)^3 - (u_i - u_{i-1})^3\right] \qquad (1)$$

For this experiment, the coupling constants $\kappa = \beta = 1$. A pure system is composed of masses $m^o = 1$, and disorder is achieved by randomly changing a number N_I of the masses to a second value. The time integration is performed using a 4-th order predictor-corrector algorithm. Further details will be given elsewhere.

The initial condition is a unit amplitude, zero-velocity sinusoidal displacement with wavelength $\lambda = 32$. Time is scaled by the harmonic frequency $\omega^o = 2\pi/\lambda$. At certain intervals, the time-dependent displacement of each mass is transformed to frequency space, and these data $u_k(x_i)$ are used to calculate the modal energy $E_\omega = E(\omega^o)$ and mass energy $E_i = E(x_i)$ by summing over masses and frequencies, respectively.

3 Results

The systems consist of $N = 256$ masses, which is a size consistent with systems used elsewhere [3]. Time is expressed as the dimensionless quantity $\omega^o t/2\pi$ which is equivalent to the number of harmonic cycles, and the modal energy is scaled to $E(\omega^o t/2\pi = 32)$. The error bars shown in the figures represent the estimated standard deviation in the mean.

The initial systems investigated contained impurity masses $M_I > m^o$. Comparisons of the effects upon modal decay as a function of either impurity mass M_I or number of impurities N_I are shown in Figs. 1 (a) and (b); the pure nonlinear system is denoted by open circles. As expected, the addition of impurities increases the initial rate at which the harmonic mode ω^o decays. However, for all the combinations of impurity mass and concentration shown, the long time decay is *slower* than for the system without impurities. Further, increases in impurity mass and impurity concentration retard the long time decay.

The long time behavior of the systems containing heavy impurities can be explained by the nonergodic behavior within the system. Systems containing impurities undergo an Anderson transition. The resulting excited modes are localized, concentrating energy near the impurities. Since the impurities are heavier than m^o, their oscillation amplitudes are smaller, resulting in a smaller nonlinear contribution to the energy at the impurity.

As a demonstration of the localization of energy at the impurities, the mass energy E_i is calculated at each impurity. The average energy at an impurity $\langle E_i\rangle_I$ is compared to the average energy at all the masses $\langle E_i\rangle_N$. The ratio $\alpha = \langle E_i\rangle_I/\langle E_i\rangle_N$, which is interpreted as a measure of the extent to which ergodicity has been achieved, is shown in Figs. 1(c) and (d).

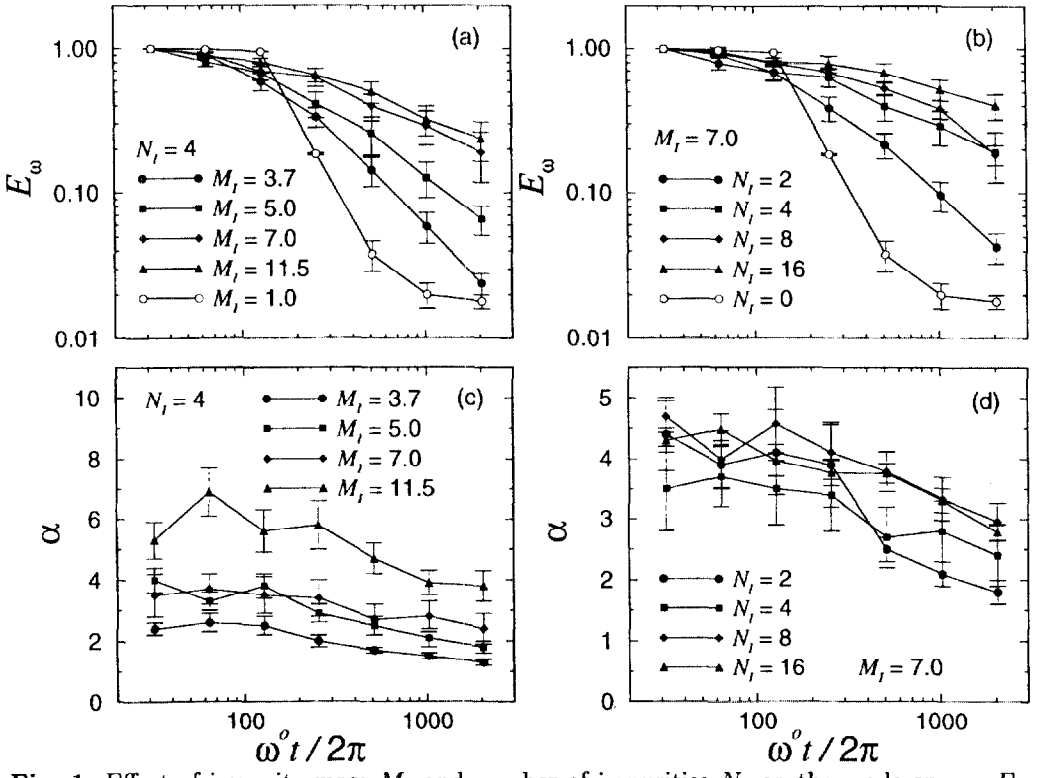

Fig. 1 Effect of impurity mass M_I and number of impurities N_I on the mode energy E_ω and relative energy α at an impurity. In (a) and (c) the number of impurities N_I is 4. In (b) and (d) the impurity mass M_I is 7. The open circles are for the nonlinear system without impurities. Error bars represent the estimated standard deviation of the mean.

Figures 1(c) and (d) show that the relative energy at an impurity increases with increasing impurity mass, and is somewhat insensitive to changes in the impurity concentration, respectively. Therefore, the total energy located at impurities is proportional to the mass and the number of impurities. This explains why the modal decay rate decreases as either the impurity mass or the impurity concentration increases.

Based upon the arguments given above, impurities with masses that are lighter than m^o should have a different effect upon mode decay. Since the impurities are lighter, the localized energy will create large oscillations, which should enhance nonlinear interactions and, hence, enhance the rate of mode decay. The results of that experiment for impurity mass $M_I = 0.1$ are shown in Fig. 2. For low concentrations, the mode decay shown in Fig. 2(a) is faster than for the pure system, as expected. However, at sufficiently high concentration, the mode decays slower than the pure system when the average spacing between impurities is one half the initial wavelength.

The relative energy α located at a light impurity shown in Fig. 2(b) is less than unity. The light impurities apparently expel energy due to the large oscillation amplitudes, hastening mode decay. Above a critical concentration, it is conjectured that the light impurities interact in such a way as to prolong the mode decay.

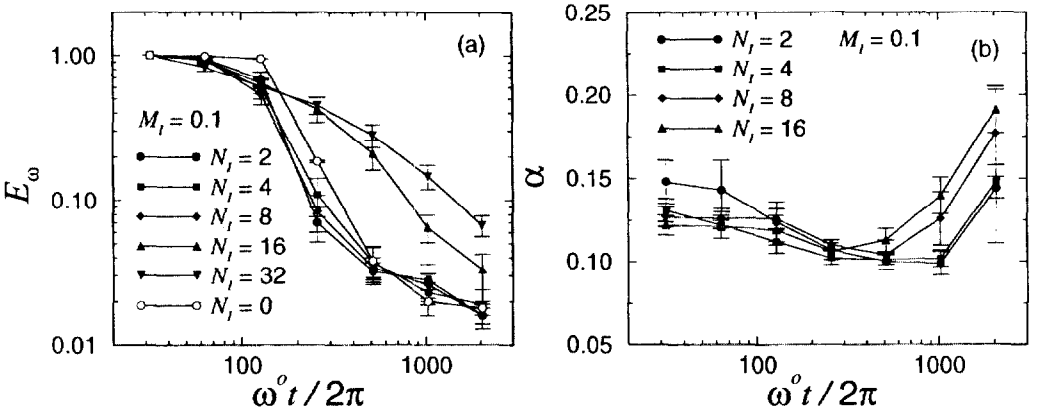

Fig. 2 The modal energy E_ω and the relative energy at the impurities α for impurities of mass $M_I = 0.1$.

4 Conclusion

For the simulation times studied here, the long time mode decay in disordered nonlinear systems seems to be controlled by disorder and Anderson localization effects. For either heavy or light impurities the energy becomes localized, and the assumption of ergodic behavior is not valid. Further, the response of the system depends upon whether the impurities are heavier or lighter than the pure system. Heavier impurities consume energy and, because of the smaller oscillation amplitudes, release their energy through nonlinear interactions very slowly. Lighter impurities appear to expel energy through large oscillation amplitudes, hastening nonlinear interactions and mode decay at low concentrations, but behave in a manner similar to heavy impurities at high concentrations, possibly due to Anderson localization effects.

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